

# COSMOLOGICAL OBSERVATIONS IN A LOCAL VOID

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## ABSTRACT

A local void in the globally Friedmann-Robertson-Walker (FRW) cosmological model is studied. The inhomogeneity is described using the Lemaître-Tolman-Bondi (LTB) solution with the spherically symmetric matter distribution based on the faint galaxies number counts. We investigate the effects this has on the measurement of the Hubble constant and the redshift–luminosity distance relation for moderately and very distant objects ( $z \approx 0.1$  and more). The results, while fully compatible with cosmological observations, indicate that if we happened to live in such a void, but insisted on interpreting cosmological observations through the FRW model, we could get a few unexpected results. For example the Hubble constant measurement could give results depending on the separation of the source and the observer.

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## 1. INTRODUCTION

It seems, particularly after the introduction of the inflationary paradigm (Guth 1981), that the isotropic and homogeneous Friedmann-Robertson-Walker (FRW) cosmological models are best suited for the description of the global structure and the evolution of the universe. However, a similar statement is not necessarily true when cosmologically moderate scales are thought of.

There exists direct observational evidence in favour of the large scale isotropy of the observed universe, namely, the COBE data confirming a high degree of isotropy of the cosmic microwave background radiation (CMBR) (Mather *et al.* 1990; Hogan 1990; Smoot *et al.* 1993; Wright *et al.* 1993). However, the observational basis for the other standard assumption made in the FRW cosmology, the homogeneity, is weaker. There exists observational evidence in favour of larger and larger structures (e.g. Lynden-Bell *et al.* 1988; Clowes and Campusano 1991; Geller and Huchra 1989).

We think that there exists sufficient observational evidence (discussed later in this paper) to support a conjecture that we may live in a relatively large underdense region embedded in a globally FRW universe. Exploring observational properties of such a model is the aim of the present paper.

The simple model presented in this paper is restricted by the two physical demands we impose on it. Firstly, to be consistent with the CMBR isotropy it has to be spherically symmetric. Secondly, the model should be very similar to an FRW one at the beginning of the expansion, but become observably different at later times. In this manner, we could retain the accomplishments of the FRW cosmology in dealing with early epochs, while gaining new freedom in modelling the more recent universe.

The work on modelling voids of the Lemaître-Tolman-Bondi type <sup>1</sup> in the expanding FRW universe has been extensive. Main results and references will be briefly discussed later. (An excellent review of the inhomogeneous cosmology exists: Krasiński 1994.) Nevertheless, the observable consequences of such a model have seldom been studied (e.g. Paczyński and Piran 1990; Moffat and Tatarski 1992).

In the following section, we briefly describe the LTB model. Section 3. consists of a concise review of the main results in modelling LTB voids, a discussion of the observational background and the simple model of a local void presented here. The description of our

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<sup>1</sup>A cosmological solution spherically symmetric about one point was first proposed by Lemaître (Lemaître 1931; 1933). However, it is usually called the Tolman-Bondi solution (Tolman 1934, Bondi 1947).

results of numerical calculations is contained in Section 4. The closing section contains a discussion and conclusions.

Throughout this paper we use units in which  $G = c = 1$ , unless stated otherwise. Moreover, we choose the cosmological constant  $\Lambda = 0$ .

## 2. MODEL

First, for the sake of notational clarity, let us recall the FRW line element:

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1)$$

with  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ .

Now, let us consider a Lemaître-Tolman-Bondi (Tolman 1934, Bondi 1947) model for a spherically symmetric inhomogeneous universe filled with dust. The line element in comoving coordinates can be written as:

$$ds^2 = dt^2 - R'^2(t, r) f^{-2} dr^2 - R^2(t, r) d\Omega^2, \quad (2)$$

where  $f$  is an arbitrary function of  $r$  only, and the field equations demand that  $R(t, r)$  satisfies:

$$2R\dot{R}^2 + 2R(1 - f^2) = F(r), \quad (3)$$

with  $F$  being an arbitrary function of class  $C^2$ ,  $\dot{R} = \partial R / \partial t$  and  $R' = \partial R / \partial r$ . We have three distinct solutions depending on whether  $f^2 < 1$ ,  $= 1$ ,  $> 1$  and they correspond to elliptic (closed), parabolic (flat) and hyperbolic (open) cases, respectively.

The proper density can be expressed as:

$$\rho = \frac{F'}{16\pi R' R^2}. \quad (4)$$

Whatever the curvature, the total mass within comoving radius  $r$  is:

$$M(r) = \frac{1}{4} \int_0^r dr f^{-1} F' = 4\pi \int_0^r dr \rho f^{-1} R' R^2, \quad (5)$$

so that

$$M'(r) = \frac{dM}{dr} = 4\pi \rho f^{-1} R' R^2.$$

Also for  $\rho > 0$  everywhere we have  $F' > 0$  and  $R' > 0$  so that in the non-singular part of the model  $R > 0$  except for  $r = 0$  and  $F(r)$  is non-negative and monotonically increasing

for  $r \geq 0$ . This could be used to define the new radial coordinate  $\bar{r}^3 = M(r)$  and find the parametric solutions for the rate of expansion.

In the flat (parabolic) case  $f^2 = 1$ , we have

$$R = \frac{1}{2}(9F)^{1/3}(t + \beta)^{2/3}, \quad (6)$$

with  $\beta(r)$  being an arbitrary function of class  $C^2$  for all  $r$ . After the change of coordinates  $R(t, \bar{r}) = \bar{r}(t + \beta(\bar{r}))^{2/3}$ , the metric becomes:

$$ds^2 = dt^2 - (t + \beta)^{4/3} (Y^2 dr^2 + r^2 d\Omega^2), \quad (7)$$

where

$$Y = 1 + \frac{2r\beta'}{3(t + \beta)}, \quad (8)$$

and from (4) the density is given by

$$\rho = \frac{1}{6\pi(t + \beta)^2 Y}. \quad (9)$$

Clearly, we have that ( $t \rightarrow \infty$ ) the model tends to the flat Einstein–de Sitter case.

For the closed and open cases the parametric solutions for the rate of expansion can be written as (Bonnor 1972; 1974):

$$R = \frac{1}{4}F(1 - f^2)^{-1} [1 - \cos(v)], \quad f^2 < 1, \quad (10a)$$

$$t + \beta = \frac{1}{4}F(1 - f^2)^{-3/2} [v - \sin(v)], \quad f^2 < 1, \quad (10b)$$

and

$$R = \frac{1}{4}F(f^2 - 1)^{-1} [\cosh(v) - 1], \quad f^2 > 1, \quad (11a)$$

$$t + \beta = \frac{1}{4}F(f^2 - 1)^{-3/2} [\sinh(v) - v], \quad f^2 > 1, \quad (11b)$$

with  $\beta(r)$  being again a function of integration of class  $C^2$  and  $v$  the parameter.

The flat case ( $f^2 = 1$ ) has been extensively studied elsewhere (Moffat and Tatarski 1992). The model depends on one arbitrary function  $\beta(r)$  and could be specified by assuming the density on some space-like hypersurface, say  $t = t_0$ . However, specifying the density on the past light cone of the observer is more appropriate.

The cases of interest to us, (10) and (11), correspond to closed and open models, respectively.

Before we proceed (in the next section) to discuss the observational grounds for modelling a local void, we need to amplify the discussion of the LTB model by introducing basic features of the propagation of light. The high degree of isotropy of the microwave background forces us to the conclusion that we must be located close to the spatial centre of the local LTB void. In our discussion, for the sake of simplicity, we place an observer at the centre ( $t_{Ob} = t_0, r_{Ob} = 0$ ).

The luminosity distance between an observer at the origin of our coordinate system ( $t_0, 0$ ) and the source at ( $t_e, r_e, \theta_e, \phi_e$ ) is (Bondi 1947):

$$d_L = \left( \frac{\mathcal{L}}{4\pi\mathcal{F}} \right)^{1/2} = R(t_e, r_e)[1 + z(t_e, r_e)]^2, \quad (12)$$

where  $\mathcal{L}$  is the absolute luminosity of the source (the energy emitted per unit time in the source's rest frame),  $\mathcal{F}$  is the measured flux (the energy per unit time per unit area as measured by the observer) and  $z(t_e, r_e)$  is the redshift (blueshift) for a light ray emitted at ( $t_e, r_e$ ) and observed at ( $t_0, 0$ ).

The light ray travelling inwards to the centre satisfies:

$$ds^2 = dt^2 - R'^2(t, r)f^{-2}dr^2 = 0, \quad d\theta = d\phi = 0,$$

and thus

$$\frac{dt}{dr} = -R'(t, r)/f(r), \quad (13)$$

where the sign is determined by the fact that the light ray travels along the *past* light cone (i.e. if  $r_{e'} > r_{e''}$ , then  $t_{e'} < t_{e''}$ ).

Without getting into a detailed discussion, which can be found in Bondi 1947, or Moffat and Tatarski 1992, let us state that if the equation of the light ray travelling along the light cone is:

$$t = T(r), \quad (14)$$

using (13), we get the equation of a ray along the path:

$$\frac{dT(r)}{dr} = -\frac{R'}{f}[T(r), r], \quad (15)$$

where

$$\dot{R}'[T(r), r] = \left. \frac{\partial^2 R}{\partial t \partial r} \right|_{r, T(r)} = \left. \frac{\partial R'}{\partial t} \right|_{r, T(r)}.$$

The equation for the redshift considered as a function of  $r$  along the light cone is:

$$\frac{dz}{dr} = (1 + z)\dot{R}'[T(r), r], \quad (16)$$

and the shift  $z_1$  for a light ray travelling from  $(t_1, r_1)$  to  $(t_0, 0)$  is:

$$\ln(1 + z_1) = -\ln(1 - a_1) - \int_0^{r_1} dr \frac{M'(r)}{r(1 - a_1)}, \quad (17)$$

where

$$a_1(r) = \dot{R}[T(r), r],$$

and, in obtaining equation (17), we used (4) and (5). Thus we have two contributions to the redshift. The cosmological redshift due to expansion, described by the first term with  $a_1 = \dot{R}$ , and the gravitational shift due to the difference between the potential energy per unit mass at the source and at the observer. Obviously, in the homogeneous case ( $M'(r) = 0$ ) there is no gravitational shift.

### 3. LOCAL VOID

If we restrict ourselves to spatial scales that have been well probed observationally, i.e. up to a few hundred Mpc, the most striking feature of the luminous matter distribution is the existence of large voids surrounded by sheet-like structures containing galaxies (e.g. Geller and Huchra 1989). These surveys (see also Efstathiou *et al.* 1990; Saunders *et al.* 1991) give a typical size of the voids of the order  $50\text{--}60 h^{-1}$  Mpc. There has also been some evidence (Broadhurst *et al.* 1990) –with less certainty– for the existence of larger underdense regions with characteristic sizes of about  $130 h^{-1}$  Mpc. Also, dynamical estimates of the FRW density parameter  $\Omega_0$  give very different results on different scales. The observations of galactic halos on scales less than about 10 to 30 Mpc typically give (see e.g. Sancisi and van Albada 1987)  $\Omega_{10\text{--}30} \simeq 0.2 \pm 0.1$ . On the other hand, smoothing the observations over larger scales ( $> 20$  Mpc, say  $\sim 100$  Mpc) indicates (e.g. Efstathiou *et al.* 1990; Saunders *et al.* 1991) the existence of a less clustered component with a contribution exceeding 0.2, and perhaps as high as  $\Omega_{\sim 100} \simeq 0.8 \pm 0.2$ .

At the same time, the large scale galaxy surveys (some of the recent literature is given in Maddox *et al.* 1990; Tyson 1988; Heydon-Dumbleton *et al.* 1989; Lilly 1993) firmly indicate a considerable excess in the number–magnitude counts for faint galaxies relative to predictions of homogeneous, “no-evolution” models. This excess could be the result of a non-standard galactic evolution or could be caused by rather exotic FRW cosmology (i.e. the deceleration parameter  $q_0 \ll 0.5$  or a non-zero cosmological constant  $\Lambda$ ). However, it can also be treated as an observational indication of a very large (on the scale of the redshift  $z \approx 0.5$ ) void. In the following, we choose this latter option in interpreting the faint galaxies number counts and model the density distribution of a local void accordingly. (A

somewhat similar model of a gaussian void as a function of the comoving coordinate can be found in Moffat and Tatarski 1994.)

We study a void with the central density equal to that of an FRW model with the density parameter  $\Omega_0 = 0.2$ , asymptotically approaching the FRW model with  $\Omega_0 = 1$ . Since cosmological observations are done by detecting some form of electromagnetic radiation, the mapping obtained from them describes the density along the light cone. We describe the density distribution, as a function of the redshift  $z$ , by:

$$\Omega_v(z) = \frac{\Omega_{min} + (z/a)^2}{\Omega_{max} + (z/a)^2}. \quad (18)$$

The choice of a rational function  $\Omega_v(z)$  (as opposed to, say, a gaussian distribution) assures that the differential equations that we solve numerically (already quite complex) are not unnecessarily complicated further. In the numerical calculations presented in the next section we used the values  $\Omega_{min} = 0.2$ ,  $\Omega_{max} = 1$ . The two density distributions presented there are parametrized by (A)  $a = 0.125$  and (B)  $a = 0.25$ , and are depicted in Figure 1.

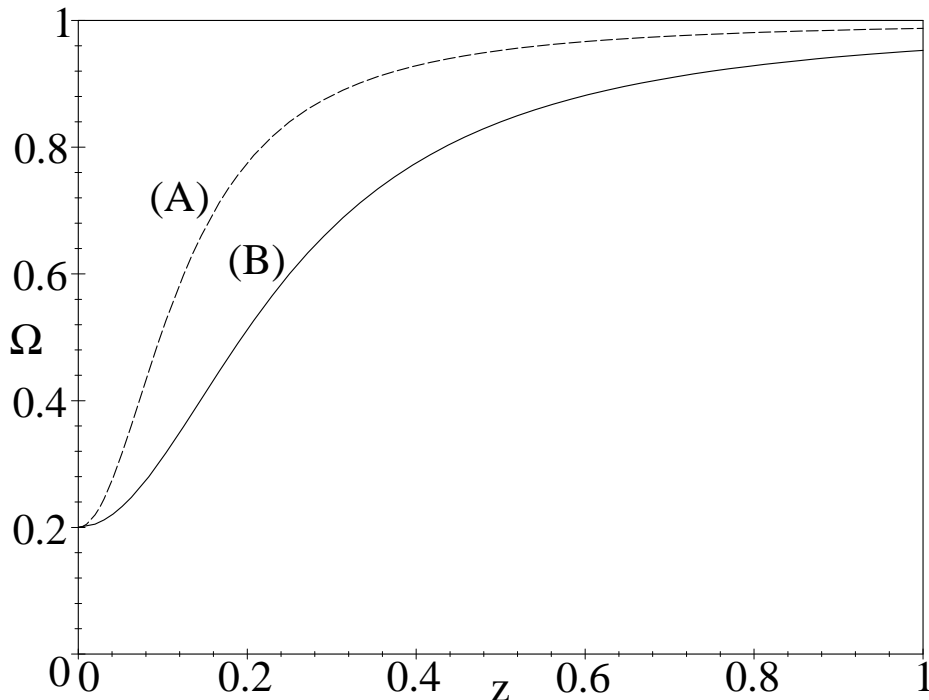


Fig. 1.— The density distributions  $\Omega_v$  as functions of the redshift  $z$ . The parametrization of the cases (A) and (B) is given in the text.

In the following figures the results obtained for the (steeper) distribution (A) will be represented by dashed lines, while the results for the density distribution (B) will always be given as solid lines. Also, since we study here the observations of the luminous matter, we

present our results in the range of up to  $z \approx 6$ . (An observation of an object with a redshift in excess of this value would be hard to reconcile with standard FRW cosmology.)

The final question we wish to address in this section is that of shell crossing in the LTB models. The problem has been extensively studied (for a detailed review and a complete list of references see Krasiński 1994).

As seen from (2) and (4) shell crossing can occur when  $R'(t, r) = 0$  for some  $r = r_s$ . This, in general, may lead to  $\lim_{r \rightarrow r_s} \rho = \infty$  and a shell crossing singularity. Different shells  $r = \text{const}$  collide and the comoving coordinates become inadmissible. The singularity can be avoided, if the functions  $F'(r)$  and  $f(r)$  both have at  $r = r_s$  zeros of the same order as  $R'(t, r)$  has. This, however, is not the case in the present work (we assume  $F(r) \geq 0$  and  $F'(r) \geq 0$ ).

In the simple model presented here, shell crossing does occur. The physical significance of this can be viewed from diametrically opposite positions. One extreme is to regard a shell crossing singularity as unphysical and try to exclude it by assuming special initial conditions, as was the strategy in the work of Bonnor and Chamorro (Bonnor and Chamorro 1990; 1991; Chamorro 1991). However, this leads to difficulties with adjusting the model to observational data. In short: either the average density of the void is not appreciably lower than the density of the background, or the matter in the void is much older than outside. The second approach, best represented by Sato and collaborators (see e.g. Sato 1984; Suto *et al.* 1984a, 1984b), is to treat the shell crossing singularity as a physical entity. In this approach, shell crossings –numerically modelled as surface layers of matter– are interpreted as rims of the expanding voids and correspond to the aforementioned sheet-like structures surrounding observed voids.

An observationally acceptable LTB void, with the density distribution similar to that depicted in Figure 1, is too deep to escape shell crossing and, at the same time, allow for the simultaneous Big Bang throughout the whole space. Since we see the latter property as very important in order to be able to incorporate the accomplishments of the FRW cosmology in dealing with epochs preceding the matter dominated era, we choose to accept shell crossing as a price. The more so, that –as will be shown in the next section– the distribution (18), even though chosen on purely observational grounds, leaves us with a bonus: the shell crossing happens in the distant future. This, in our opinion, indicates that the LTB void studied here is applicable to the matter dominated era. (The fact that the LTB model does not allow for pressure is not perilous from our point of view. After all, the universe has been matter dominated since  $z \approx 10^4$ . The model is applicable to this era and may be thought of as a continuation of an earlier –very nearly– FRW model, provided its density contrast at the time of decoupling,  $t = t_{dc}$ , is  $\rho(t_{dc}, r)/\rho_{FRW}(t_{dc}) \approx 1$ .)



## 4. RESULTS

In general, an LTB model depends on three arbitrary functions (see section 2.),  $F(r)$ ,  $\beta(r)$  and  $f(r)$ . Since  $F(r)$  can be interpreted as twice the effective gravitational mass within comoving radius  $r$  (Bondi 1947), then, in accordance with the discussion following (5), assuming its form is equivalent to a coordinate choice. In our calculations we used  $F(r) = 4r^3$ . The second function,  $\beta(r)$ , sets the initial singularity hypersurface of the model. Since we want the region of highest density in our model to be fully equivalent to the FRW universe, we set  $\beta(r) = \beta_0 = \text{const}$ , thereby assuming a universally simultaneous Big Bang. In doing so, we give up a very important feature of an LTB model: an extra (with respect to FRW) degree of freedom that would allow the age of the universe to be position dependent. However, we study the following two cases (both with the simultaneous Big Bang hypersurface): the universe whose age is equal to that of the FRW critical ( $\Omega_0 = 1$ ) case and the universe with the age equal to that of the FRW  $\Omega_0 = 0.2$  one. The parametrization of the cases is, respectively, (I)  $\beta_0 = \beta_I = 0$  and (II)  $\beta_0 = \beta_{II} = 1.23606$ , where –in both cases– we set the time coordinate of constant time hypersurface “now” so that it is equal to the age of the universe  $t_0 = 1$  in the FRW model with  $\Omega_0 = 1$ . The third (“curvature”) function,  $f(r)$ , is an unknown to be solved for in our calculations. Since we are modelling an underdense comoving void in an FRW universe, we choose the LTB hyperbolic ( $f^2 > 1$ ) case.

In a manner similar to that employed in Moffat and Tatarski 1992, we assume that since all cosmological observations are necessarily done by detecting some form of electromagnetic radiation, the solution should progress along the light cone. The final set of equations we solve consists of the equations (15), (16) and the equation describing the ratio of the local density (4) to the density of the FRW universe through the relation (18) taken along the past light cone:

$$\frac{\rho[T(r), r]}{\rho_{FRW}[T(r)]} = \frac{3F'(r)[T(r) + \beta_0]^2}{8R'[T(r), r]R^2[T(r), r]} = \Omega_v[z(r)]. \quad (19)$$

The final form of the equations is quite unwieldy and we do not present them here. Since  $T(r)$  (the time of emission  $t_e$  of a light ray observed at  $r = 0$  at  $t_0$ ) is now given by (11b) and  $z(r)$  is governed by (16), the functions to be solved for are  $f(r)$ ,  $z(r)$  and  $v(r)$ , where the parameter  $v$  becomes the function of position.

The initial conditions for the integration have to be set at  $r \neq 0$ , since the analytic expressions (11) are singular at  $r = 0$ , where  $f^2 = 1$  (we have a flat  $\Omega_0 = 0.2$  FRW universe there). We assume that for the initial radius  $r_i \ll 1$  (we use dimensionless radius and time in the calculations) corresponding  $z_i$  and  $t_i$  are given by their standard FRW values. Then  $z(r_i)$ ,  $v(r_i)$  and  $f(r_i)$  can be obtained from (11b), (15) and (16).

Once the equations have been numerically integrated (in all cases the accuracy of our numerical procedure was  $10^{-6}$  or better <sup>2</sup>) we use (11b) to obtain  $t_e$  for a given  $r_e$ . The luminosity distance  $d_L$  corresponding to this event is obtained with the use of (12) and  $z(t)$  (useful in studying the cosmological time scale) is given by the parametric relation  $[T(r), z(r)]$ .

The results of our numerical calculations for  $z(r)$ , where  $r$  is the dimensionless comoving radius used in the calculations are depicted in Figure 2. The coordinate distance has no direct physical relevance, but our units here are such that  $r = 1$  corresponds to  $2997.95h^{-1}$  Mpc, where  $h$  is the usual coefficient in the observationally determined value of the Hubble constant:  $H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1}$ .

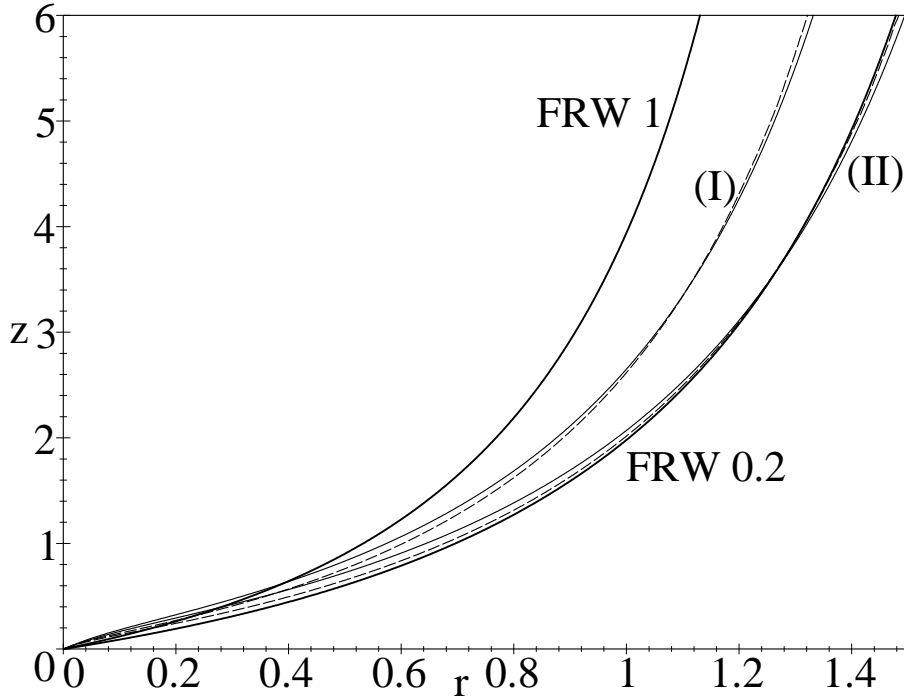


Fig. 2.— The redshift  $z$  as a function of the comoving radius  $r$ . Roman numerals I and II denote the “younger” ( $\beta_0 = \beta_I$ ) and “older” ( $\beta_0 = \beta_{II}$ ) solutions, respectively. Dashed and solid lines correspondence to the two density distributions of the void is the same as in Figure 1. FRW  $\Omega = 0.2$  and  $\Omega = 1$  solutions, respectively, are denoted “FRW 0.2” and “FRW 1”.

The behaviour of  $z(r)$  is as intuitively expected. In the “older” case (II) the redshift  $z$  asymptotically converges to the FRW  $\Omega_0 = 0.2$  behaviour. The additional gravitational

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<sup>2</sup>We do not elaborate here on the intricacies of the numerical procedure. A good discussion on the technical aspects of numerically solving a somewhat similar problem involving an LTB model can be found in Paczyński and Piran 1990.

shift caused by the mass distribution of the void is apparent on the scales comparable to that of the void ( $z \lesssim 0.5$ ). On intermediate to large scales the effect of a cosmological expansion is predominant. In the “younger” case (I),  $z(r)$  –after an excess over the FRW relation due to the gravitational shift induced by the void– asymptotically tends to a limit that could, in accordance with the FRW interpretation, correspond to the universe with the density parameter in the range  $\Omega \in (0.2; 1)$ . In both instances, the convergence to the FRW relation is marginally faster for the cases with steeper density distributions.

However, one should not forget that the comoving distance is *not* an observable, whereas the luminosity distance  $d_L$  is. In principle, provided we know its absolute luminosity  $\mathcal{L}$ , we can establish the luminosity distance, defined by  $4\pi d_L^2 = \mathcal{L}/\mathcal{F}$ , by measuring the energy flux  $\mathcal{F}$  of an observed object (for a discussion of usual caveats associated with so-called “standard candles” see e.g. Weinberg 1972; Kolb and Turner 1990; Peebles 1993). It is the  $z(d_L)$  relation that is observationally relevant. Let us recall that the FRW equivalent of (12) is:

$$d_L^2 = a^2(t_0) r_e^2 (1 + z_e)^2. \quad (20)$$

Before we probe the observable properties of the solution, let us investigate the temporal evolution of the void. In doing so we seek answers to two important questions. First, how soon in the further evolution of the void the formation of a shell crossing singularity occurs. If this happens in the distant enough future, then our model is applicable to a sufficiently long cosmological epoch to be considered physical. Secondly, we want to verify that at early times the density contrast  $\rho_v/\rho_b \approx 1$ , where  $\rho_v$  and  $\rho_b$  are matter densities in the void and in the background, in accordance with our conjecture that the model be very similar to an FRW one at the beginning of the expansion.

In general, the earlier the formation time of the void, the faster the growth of the density contrast. Also, the rate of growth is increased in the cases of higher initial contrasts. (See Sato 1984. A very thorough numerical study of the evolution of general relativistic voids, including the LTB ones, can also be found in Vadas 1993.) In the study presented here, however, we did not assume a density distribution on some hypersurface of constant time as initial condition. The observationally based density distributions of Figure 1 were given as functions of the redshift  $z$  and since we numerically integrated the equations along the light cone, we are not yet in the position to examine the time evolution of the spatial distribution of the matter density. However, equipped with the solution for  $f(r)$  we can use (11b) to obtain the values of the parameter  $v_h(r)$  corresponding to a given hypersurface of constant time  $t = t_h$  throughout the space. Then, using (4) we get the spatial matter density distributions for the time  $t_h$ .

The results (numerical solution points) of this calculation for the future and past

evolution of the steeper void (A) in the “younger” case (I) are presented in Figures 3 and 4, respectively. The density distribution in this case evolves fastest and, as such, is the most representative in estimating the time scales involved.

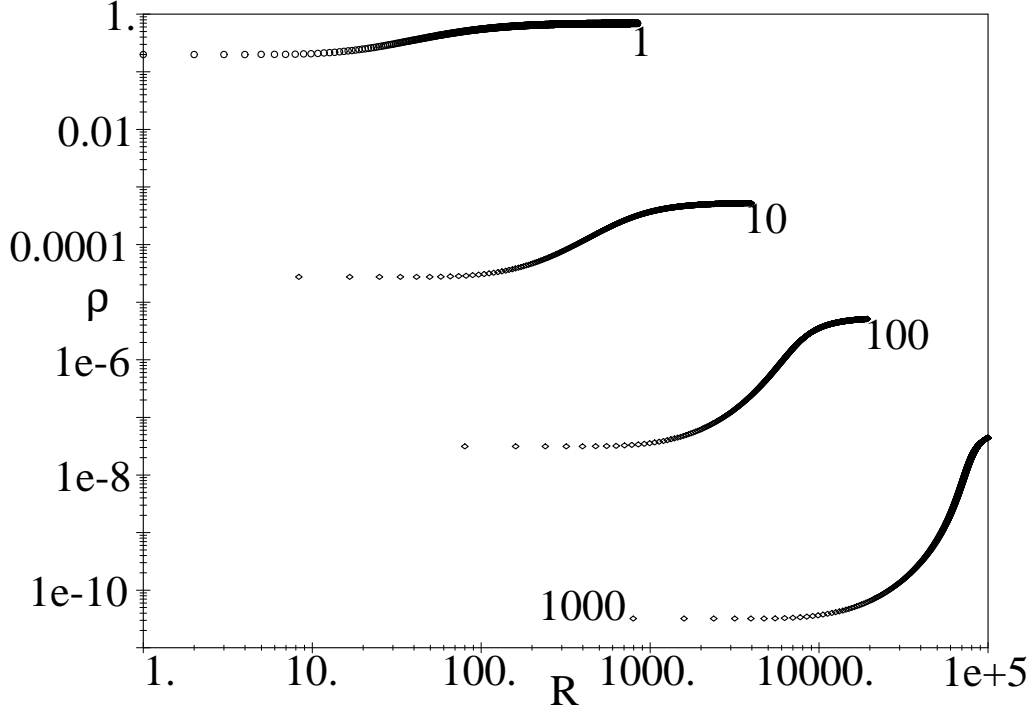


Fig. 3.— The future evolution of the void (A) in the case (I):  $t_0 + \beta_I = t_0 = t_{FRW1}$ . The matter density as a function of  $R(t_h, r)/R(t_0, r_i)$ , where  $r_i$  is the initial radius, is shown at times  $t_h = t_0, 10t_0, 100t_0$  and  $1000t_0$ .

Figure 3 shows that the void expands outward increasing its density contrast and forming a very steep ridge on its boundary. This (in accordance with Sato 1984; Vadas 1993) leads to the eventual formation of a thin dense shell and the shell crossing singularity. Of interest to us, however, is the time scale of this process. Since the shell crossing occurs for  $t \gtrsim 1000t_0$ , where  $t_0$  is the present age of the universe, we conclude that our model is applicable to a satisfactorily long cosmological epoch.

At the same time, Figure 4 clearly indicates that our model satisfies the requirement of similarity to the FRW one at early cosmological times. At  $t_h = 0.001t_0$  the matter density is approximately constant everywhere. For the case depicted in Figure 4 the density contrast is  $\rho_v/\rho_b \approx 0.982$  for  $t_h = 0.001t_0$  ( $\approx 0.996$  for  $t_h = 10^{-4}t_0$ ). In all cases at  $t/(t_0 + \beta_0) \simeq 10^{-6}$  we have  $|\rho_v/\rho_b - 1| \simeq 10^{-5}$ . There is hardly a trace of the void’s presence.

The  $z(t)$  relations for the cases of different “ages of the universe” are presented in Figures 5 and 6. There is some divergence from the FRW behaviour, but for early

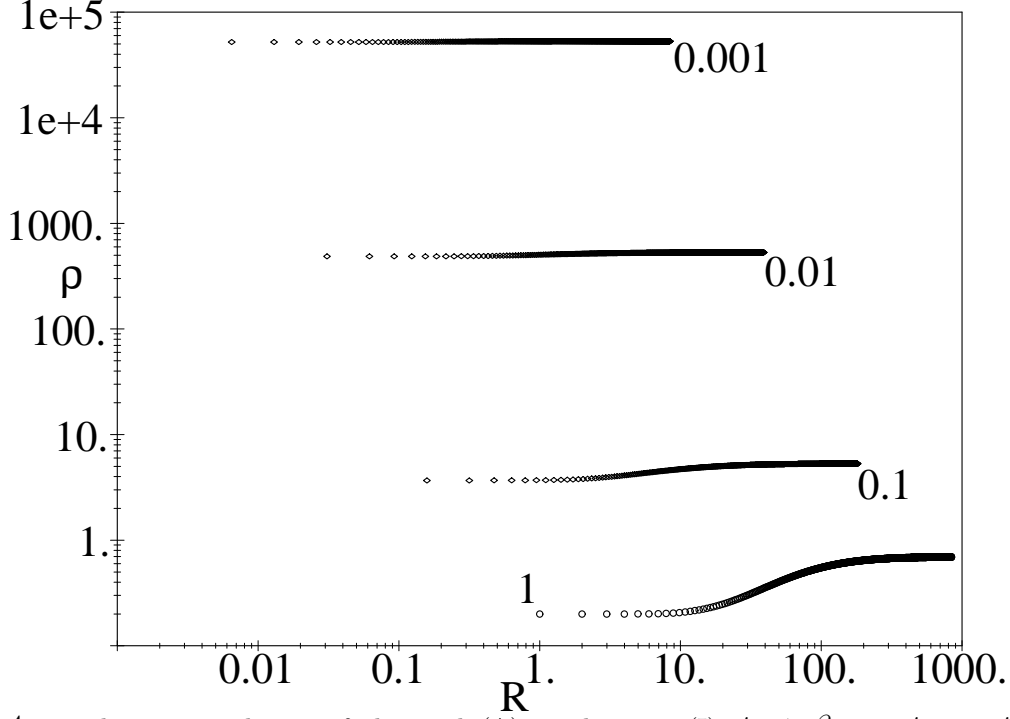


Fig. 4.— The past evolution of the void (A) in the case (I):  $t_0 + \beta_I = t_0 = t_{FRW1}$ . The matter density as a function of  $R(t_h, r)/R(t_0, r_i)$ , where  $r_i$  is the initial radius, is shown at times  $t_h = t_0, 0.1t_0, 0.01t_0$  and  $0.001t_0$ .

cosmological times the relations tend to their FRW counterparts. This asymptotic behaviour is in accordance with our assumption of a simultaneous big bang,  $\beta(r) = \text{const}$ , and with our setting the age of the universe in each case to be equal to that of the corresponding FRW case. In both (I) and (II) cases objects with similar redshifts are younger than their FRW counterparts (i.e.  $t_{LTB} > t_{FRW}$ , when  $z_{LTB} = z_{FRW}$ ), but not significantly. Those objects have marginally more time to form and evolve than their FRW counterparts.

Let us now explore the  $z(d_L)$  relations resulting from our model. Any nonstandard cosmological model to be of interest must pass two tests when confronted with observations. To be considered realistic, it is of utmost importance that its predictions do not contradict the linearity of Hubble’s law  $z = Hd$ , well established on small scales. At the same time, it should be sufficiently different from the standard (FRW) models to allow for significant reinterpretation of observational results. Let us now apply these two tests to our model.

Figure 7 shows that (in all cases studied here) on large scales the LTB “Hubble relation” between the redshift  $z$  and the luminosity distance  $d_L$  is reminiscent of the FRW relation for the universe with the density parameter in the range  $\Omega \in (0.2; 1)$ .

At the same time, inspection of Figure 8 shows that on small scales very nearly linear

“Hubble diagrams” are obtained for both types of LTB voids and both time scales.

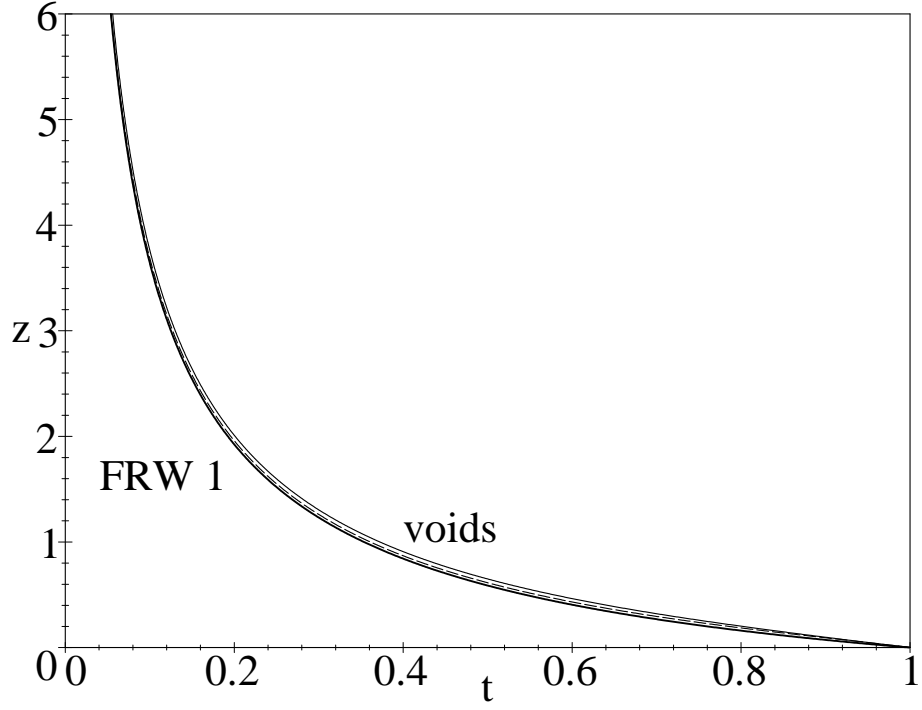


Fig. 5.— The redshift  $z$  as a function of the cosmic time  $t$  for the case (I):  $t_0 + \beta_I = t_{FRW1}$ . ( $t = 1$  corresponds to the  $t_0$  in the FRW critical case.)

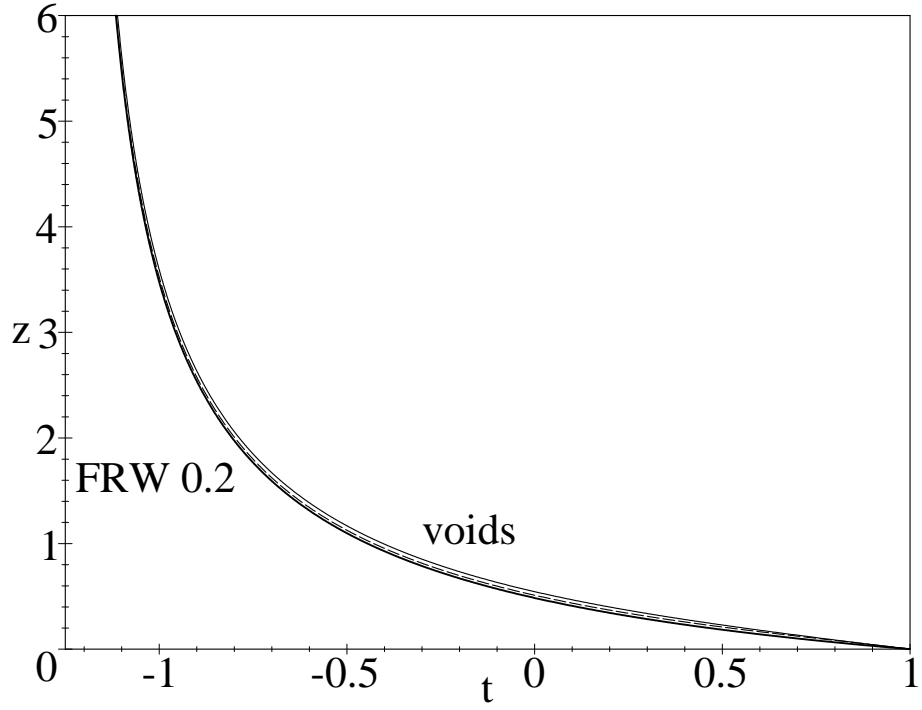


Fig. 6.— The redshift  $z$  as a function of the cosmic time  $t$  for the case (II):  $t_0 + \beta_{II} = t_{FRW0.2}$ . ( $t = 1$  corresponds to the  $t_0$  in the FRW critical case.)

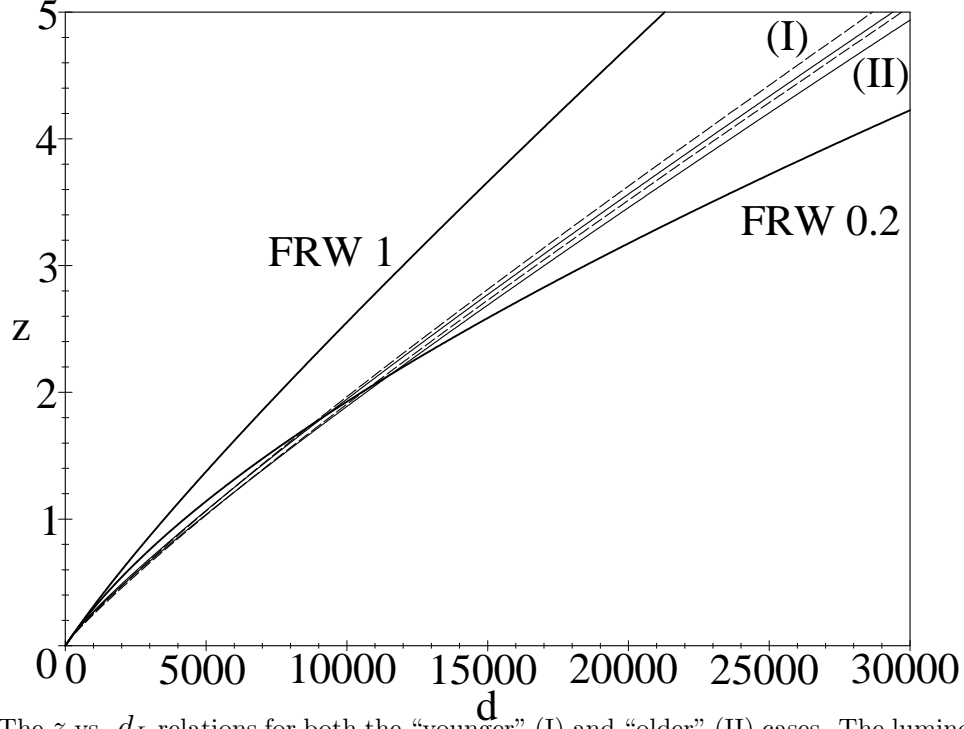


Fig. 7.— The  $z$  vs.  $d_L$  relations for both the “younger” (I) and “older” (II) cases. The luminosity distance  $d_L$  is given in Mpc.

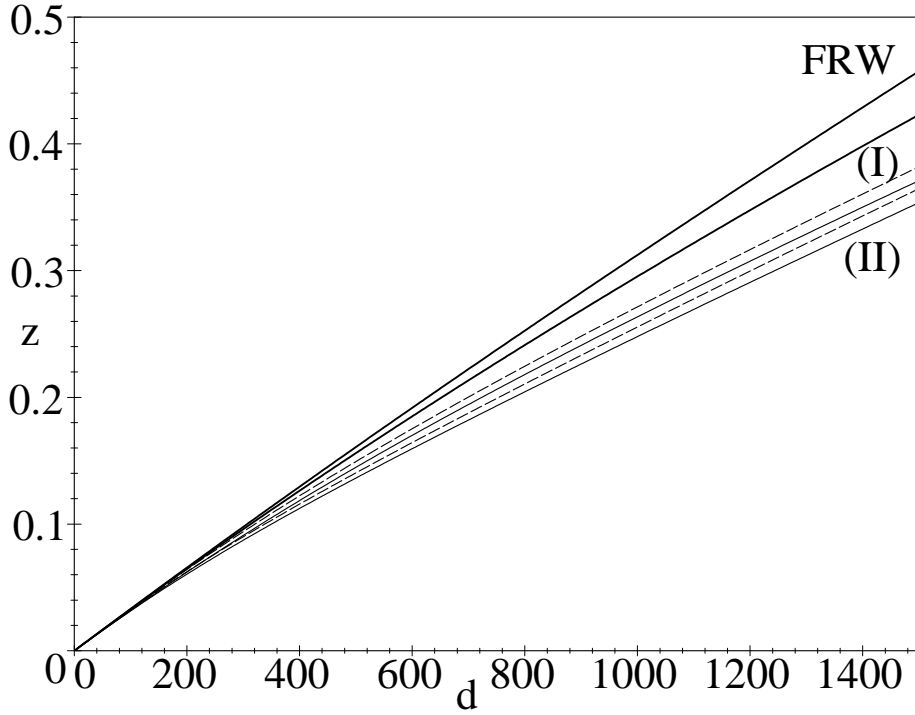


Fig. 8.— The  $z$  vs.  $d_L$  relations for both the “younger” (I) and “older” (II) cases for smaller scales. The luminosity distance  $d_L$  is given in Mpc.



If we recall (Sandage 1972, a good overview can be found in §5 of Peebles 1993) that the Hubble diagram, extended to  $z \approx 0.5$ , follows the Hubble’s law with the redshifts’ scatter (assuming identical absolute magnitudes of observed galaxies) of  $\sim 20\%$  around the theoretical prediction, then we can claim that the redshift–distance (redshift–magnitude) relations inferred from our model are within the observational bounds. Also, inspection of Figure 8 shows that on small scales observationally indistinguishable from linear “Hubble diagrams” are obtained. However, a different value for the Hubble parameter (constant) is inferred, if we insist on interpreting the results of cosmological observations through an FRW model.

Also for larger observed redshifts (see e.g. the results of McCarthy’s survey of the state of the measurements of the infrared Hubble diagram for radio galaxies, out to  $z \approx 4$ , in §5 of Peebles 1993) our results are well within the observational scatter. Moreover, when the measurements of the Hubble diagram are extended to large scales an interesting feature emerges. The slope of the diagram at  $z \lesssim 1$  seems to be consistent with the linear Hubble law. However, at  $z \gtrsim 1$  a hint of curvature appears. In the standard FRW interpretation, this is presumed to be the result of relativistic corrections to the redshift–distance relation and of the fact that the galaxies observed at high redshifts are seen as they were younger and very likely more luminous than the galaxies observed at low redshifts.

The fact that the redshift is observed to increase with distance, at least roughly in accordance with Hubble’s law, for redshifts out to in excess of unity, is usually heralded as one of the major observational confirmations of standard FRW cosmology. It would be interesting to see how the predictions of our model withstand the same observational test.

Figure 9 depicts the logarithmic redshift–luminosity distance relations for both LTB voids and both time scales. A survey of measurements (adapted from Lilly 1994) is also included. The  $z(d_L)$  relations for the FRW cases with  $\Omega_0 = 1$  and  $\Omega_0 = 0.2$  are presented for comparison.

It is now apparent that the redshift–luminosity distance relations inferred from our model are in agreement with observations. For small redshifts, say up to  $z \approx 0.1$ , they are observationally indistinguishable from the FRW results. For intermediate redshifts –in the range, say,  $z \in (0.1; 1)$ –, where the gravitational shift contribution to the overall redshift causes the most manifest departure from the FRW relations, our results remain well within the observational scatter. In the range  $z \gtrsim 1$ , where the “hint of curvature” in the redshift–distance relation is observed, our results fit the observations better than the FRW  $\Omega = 1$  prediction. However, a still better fit is provided by the FRW  $\Omega = 0.2$  curve.

Due to the possible (e.g. evolutionary) corrections to the redshift–distance relation

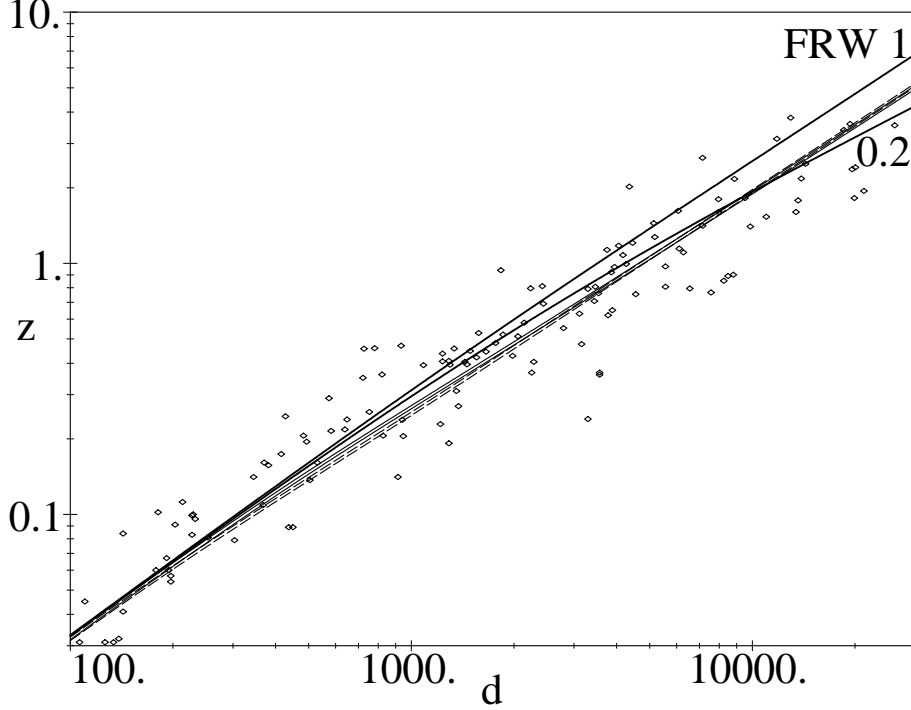


Fig. 9.— The  $\log(z)$  vs.  $\log(d_L)$  relations. Observations, denoted by  $\diamond$ , are adapted from Lilly 1994. The luminosity distance  $d_L$  is given in Mpc.

and the minute dissimilarities between FRW and LTB relations no assertions as to the best fit to observations seem irrefutable. Nevertheless, it is worth noticing that the model presented here supplies us with means to reinterpret the correlation between the theory and observational results (perhaps improving agreement between the two).

As a final element of the discussion of observational properties of our model with a local LTB void, let us concentrate on the Hubble parameter (constant) measurement. In a spherically symmetric model we have two “Hubble parameters”:  $H_r(t, r)$  for the local expansion rate in the radial direction and  $H_\perp(t, r)$  for expansion in the perpendicular direction. Usual definitions give:

$$H_r = \frac{\dot{l}_r}{l_r} = \frac{\dot{R}'}{R'}, \quad (21a)$$

$$H_\perp = \frac{\dot{l}_\perp}{l_\perp} = \frac{\dot{R}}{R}, \quad (21b)$$

where  $l$  denotes the proper distance, i.e.  $l_r = R'(t, r)f^{-1}dr$  and  $l_\perp = R(t, r)d\Omega$ . Due to the fact that there are both gravitational and expansion redshifts contributing to the total  $z$ , neither of the Hubble parameters  $H_r$ ,  $H_\perp$  is fully analogous to the FRW’s  $H_{FRW} = \dot{a}/a$ . The closest analogy exists for small separations of the source and the observer. Let us

notice that the integral in (17) can be rewritten as:

$$\int_0^{r_1} dr \frac{4\pi\rho r}{(1-a_1)^2},$$

and for small  $r_1$  can be neglected when compared to the first term of (17). Expanding the logarithms on both sides we see that light emitted at  $(t_e, r_e)$  and observed at  $(t_0, 0)$  satisfies for small  $r_e$  or small  $(t_0 - t_e)$ :

$$z(t_e, r_e) = \dot{R}(t_e, r_e),$$

where  $t_e$  is  $T(r_e)$  from (15) with the initial condition  $T(0) = t_0$ . Using (21b) and (12) we get for small  $r_e$ :

$$z(t_e, r_e) = H_\perp(t_e, r_e)d_L(t_e, r_e), \quad (22)$$

which is formally analogous to the FRW result. Two main differences are that our relation is *local* and that from cosmological observations (on small scales) we obtain the angular Hubble parameter  $H_\perp = \dot{R}/R$  rather than  $H_{FRW} = \dot{a}/a$ .

In general, if we lived in a local LTB void and the  $z$  vs.  $d_L$  relation differed from the FRW one as described in this paper, but we were biased by our theoretical prejudice and interpreted cosmological observations through an FRW model, we would expect the value of the Hubble parameter to be position, or rather  $d_L$ , dependent.

To explore this possibility let us recall that in FRW cosmology the exact result for the Hubble relation ( $z$  versus  $d_L$ ) in the matter dominated universe is (Weinberg 1972; Kolb and Turner 1990):

$$H_0 d_L = q_0^{-2} \left[ z q_0 + (q_0 - 1) \left( \sqrt{2z q_0 + 1} - 1 \right) \right], \quad (23)$$

where  $q_0 \equiv -\ddot{a}(t_0)/a(t_0)H_0^2$  is the deceleration parameter.

On cosmologically very small distances we measure the same value of  $H_0$  independently of the model (we call this value “the local measurement”). This stems from the fact that, due to our assumptions, very close to the centre ( $r \ll 1$ ) the model is well approximated by the FRW universe with  $\Omega = 0.2$ . Obviously, if the universe were locally LTB rather than FRW, then the Hubble parameter based on the observed (LTB) values of  $z$  and  $d_L$ , but inferred through an FRW relation (23), would be position (redshift) dependent. The dependence of the Hubble parameter  $H$  (in units of the  $H_0$  value as measured locally) on the redshift  $z$  is shown in Figure 10. For the sake of the clarity of presentation, only the results for the “younger” case (I) are included in the figure. Qualitatively, the “older” case (II) exhibits the same behaviour. Quantitatively, as seen from the  $z$  vs.  $d_L$  relations of Figure 7 and (23), the “observed” values of the Hubble parameter (in the units of the local

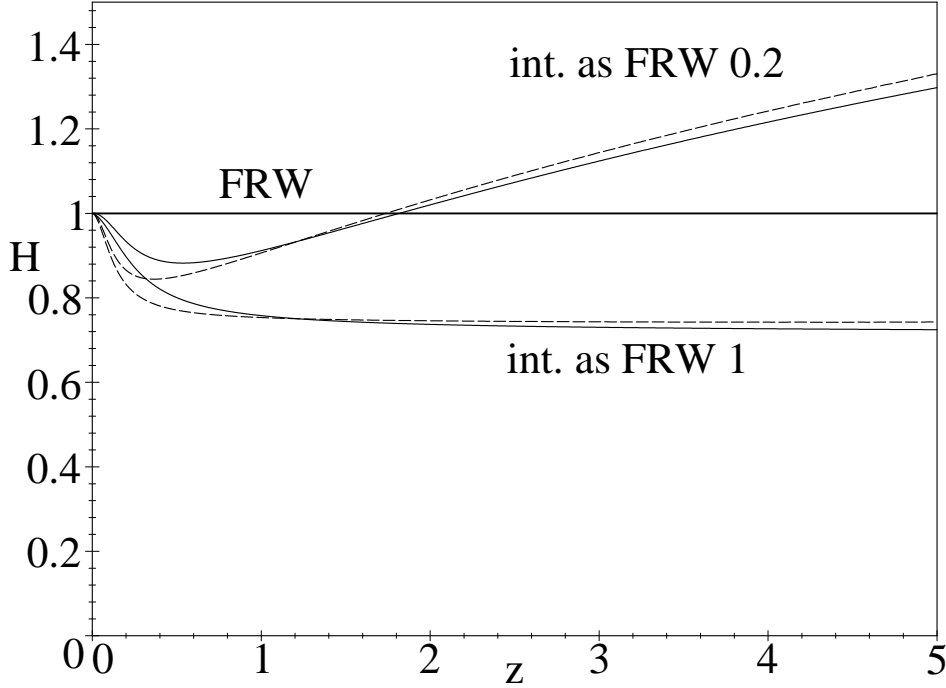


Fig. 10.— The “observed” Hubble constant  $H$  (in units of the local measurement) as a function of the redshift  $z$ .

measurement) for the case (II) become increasingly smaller than those of the case (I) ( $\approx 5\%$  smaller for  $z \approx 4$ ).

The  $H(z)$  relation depicted in Figure 10 strongly depends on the choice of the FRW case –the choice of the deceleration parameter  $q_0$  in (23)– through which we interpret the observations. The main feature in the FRW  $\Omega = 1$  interpretation is a distinct monotonic correlation between  $H$  and  $z$  (or  $d_L$ ). The values of the “observed” Hubble constant decrease with the redshift  $z$  asymptotically approaching the background limit. If we interpret the results within the FRW  $\Omega < 1$  framework, the “observed” values of the Hubble constant first decrease with  $z$  and then asymptotically increase to some background limit. (This asymptotic behaviour becomes clearly visible only for very large  $z$ .) The position of the minimum in  $H$  depends on the size of the LTB void. Neither of these patterns is evident in the reported measurements of the Hubble constant.

However, one has to remember that the measurements of the Hubble constant come inseparable from their own observational uncertainties (estimates of the distance using a distance ladder, corrections for peculiar motion etc.). Moreover, it is not the broader idea of a local LTB void embedded in a globally FRW universe that fails here. A choice of density distributions  $\Omega(z)$  more complex than (18) could result in a better agreement with observations.

We think that the variance in  $H(z)$  exhibited in Figure 10 is a valuable feature of the model. The ratios of the highest to the lowest value of the “observed”  $H$  within the range of redshifts of Figure 10 are  $\approx 1.5$ . At the same time, the values of  $H_0$  reported to date span the range 40 to 100 km s<sup>-1</sup>Mpc<sup>-1</sup> (with standard errors quoted frequently as 10 km s<sup>-1</sup>Mpc<sup>-1</sup> or less!). Inhomogeneities similar to the LTB void presented here might provide an explanation for this.

## 5. DISCUSSION

The LTB voids presented here decrease their density contrasts (the depth of the void with respect to the FRW background) when evolved back in time. At early times they are almost homogenized: at  $t/(t_0 + \beta_0) \simeq 10^{-6}$  we have  $|\rho_{LTB}(r)/\rho_{FRW} - 1| \simeq 10^{-5}$  everywhere. This corresponds to a universe which at the beginning is very similar to the FRW one, but different at late stages.

In a model utilizing a local LTB void, while retaining the accomplishments of the FRW cosmology in dealing with epochs preceding the matter dominated era, we can gain new freedom in modelling the more recent universe. We can solve the age of the universe problem by assuming  $\beta(r) = \text{const} \neq 0$ , provide the excess power observed on scales of 5–10,000 km s<sup>-1</sup> in modelling structure formation (see Moffat and Tatarski 1992) and provide an explanation for the wide range of reported values of the Hubble constant.

The use of a locally inhomogeneous model enriches the spectrum of available interpretations. The ultimate question to be addressed, however, is the agreement between predictions of the model and observations. As has been demonstrated by the results of the present work, the observationally based density distributions spanning the range of  $\Omega \in (0.2; 1)$  satisfy the observational tests. The agreement with observations is achieved both in the case of the model with the “age of the universe” equal to that of the critical ( $\Omega_0 = 1$ ) FRW one and the “older” case with  $t_0 + \beta_0$  equal to the FRW  $\Omega_0 = 0.2$  value. Theoretical prejudice (as powerful as the inflationary paradigm) might favour the critical FRW model.

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